6

Permeability

Soils are permeable due to the existence of interconnected voids through which water can flow from points of high energy to points of low energy. The study of the flow of water through permeable soil media is important in soil mechanics. It is necessary for estimating the quantity of underground seepage under various hydraulic conditions, for investigating problems involving the pumping of water for underground construction, and for making stability analyses of earth dams and earth-retaining structures that are subject to seepage forces.

6.1 Bernoulli's Equation

From fluid mechanics, we know that, according to Bernoulli's equation, the total head at a point in water under motion can be given by the sum of the pressure, velocity, and elevation heads, or

$$h = \underset{\substack{\uparrow \\ \text{Pressure} \\ \text{head}}}{\frac{u}{\gamma_w}} + \underset{\substack{\uparrow \\ \text{Velocity} \\ \text{head}}}{\frac{v^2}{2g}} + \underset{\substack{\uparrow \\ \text{Elevation} \\ \text{head}}}{Z} \tag{6.1}$$

where h = total head
u = pressure
v = velocity
g = acceleration due to gravity
γ_w = unit weight of water

Note that the elevation head, Z, is the vertical distance of a given point above or below a datum plane. The pressure head is the water pressure, u, at that point divided by the unit weight of water, γ_w.

If Bernoulli's equation is applied to the flow of water through a porous soil

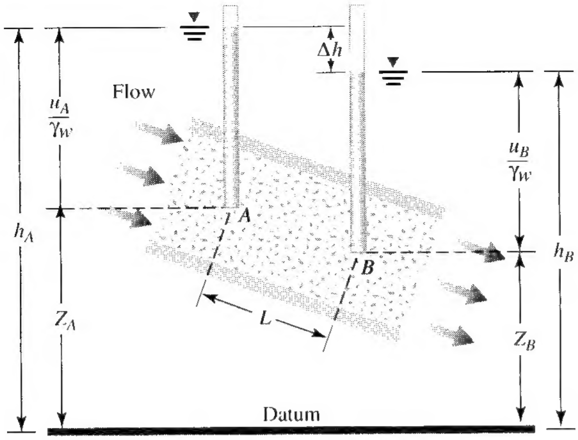

Figure 6.1 Pressure, elevation, and total heads for flow of water through soil

medium, the term containing the velocity head can be neglected because the seepage velocity is small, and the total head at any point can be adequately represented by

$$h = \frac{u}{\gamma_w} + Z \tag{6.2}$$

Figure 6.1 shows the relationship among pressure, elevation, and total heads for the flow of water through soil. Open standpipes called *piezometers* are installed at points A and B. The levels to which water rises in the piezometer tubes situated at points A and B are known as the *piezometric levels* of points A and B, respectively. The pressure head at a point is the height of the vertical column of water in the piezometer installed at that point.

The loss of head between two points, A and B, can be given by

$$\Delta h = h_A - h_B = \left(\frac{u_A}{\gamma_w} + Z_A\right) - \left(\frac{u_B}{\gamma_w} + Z_B\right) \tag{6.3}$$

The head loss, Δh, can be expressed in a nondimensional form as

$$i = \frac{\Delta h}{L} \tag{6.4}$$

where i = hydraulic gradient
L = distance between points A and B—that is, the length of flow over which the loss of head occurred

In general, the variation of the velocity v with the hydraulic gradient i is as shown in Figure 6.2. This figure is divided into three zones:

1. Laminar flow zone (Zone I)
2. Transition zone (Zone II)
3. Turbulent flow zone (Zone III)

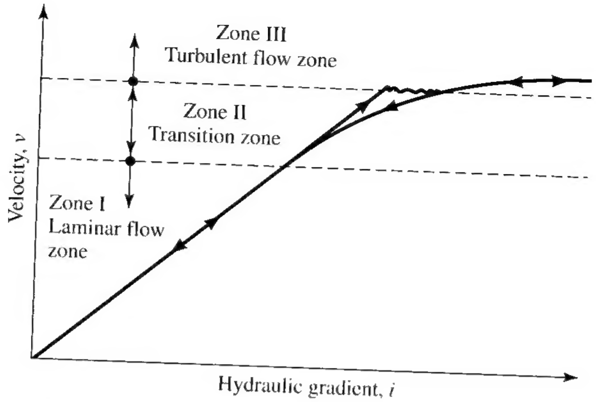

Figure 6.2 Nature of variation of v with hydraulic gradient, i

When the hydraulic gradient is gradually increased, the flow remains laminar in Zones I and II, and the velocity, v, bears a linear relationship to the hydraulic gradient. At a higher hydraulic gradient, the flow becomes turbulent (Zone III). When the hydraulic gradient is decreased, laminar flow conditions exist only in Zone I.

In most soils, the flow of water through the void spaces can be considered laminar; thus,

$$v \propto i \tag{6.5}$$

In fractured rock, stones, gravels, and very coarse sands, turbulent flow conditions may exist, and Eq. (6.5) may not be valid.

6.2 Darcy's Law

In 1856, Darcy published a simple equation for the discharge velocity of water through saturated soils, which may be expressed as

$$v = ki \tag{6.6}$$

where v = *discharge velocity*, which is the quantity of water flowing in unit time through a unit gross cross-sectional area of soil at right angles to the direction of flow

k = hydraulic conductivity (otherwise known as the coefficient of permeability)

This equation was based primarily on Darcy's observations about the flow of water through clean sands. Note that Eq. (6.6) is similar to Eq. (6.5); both are valid for laminar flow conditions and applicable for a wide range of soils.

In Eq. (6.6), v is the discharge velocity of water based on the gross cross-sectional area of the soil. However, the actual velocity of water (that is, the seepage velocity)

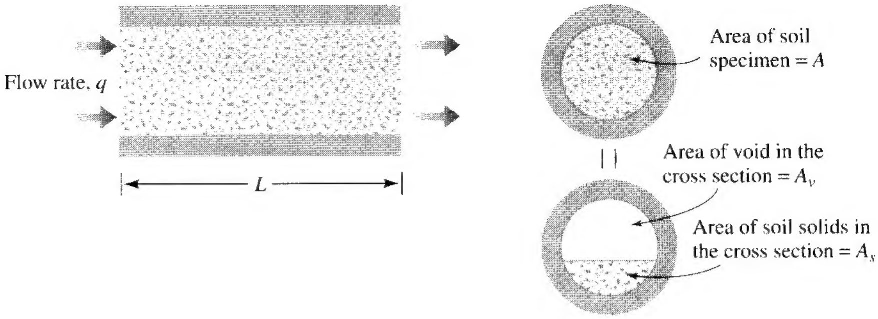

Figure 6.3 Derivation of Eq. (6.10)

through the void spaces is greater than v. A relationship between the discharge velocity and the seepage velocity can be derived by referring to Figure 6.3, which shows a soil of length L with a gross cross-sectional area A. If the quantity of water flowing through the soil in unit time is q, then

$$q = vA = A_v v_s \tag{6.7}$$

where v_s = seepage velocity
A_v = area of void in the cross section of the specimen

However,

$$A = A_v + A_s \tag{6.8}$$

where A_s = area of soil solids in the cross section of the specimen.

Combining Eqs. (6.7) and (6.8) gives

$$q = v(A_v + A_s) = A_v v_s$$

or

$$v_s = \frac{v(A_v + A_s)}{A_v} = \frac{v(A_v + A_s)L}{A_v L} = \frac{v(V_v + V_s)}{V_v} \tag{6.9}$$

where V_v = volume of voids in the specimen
V_s = volume of soil solids in the specimen

Equation (6.9) can be rewritten as

$$v_s = v\left[\frac{1 + \left(\dfrac{V_v}{V_s}\right)}{\dfrac{V_v}{V_s}}\right] = v\left(\frac{1 + e}{e}\right) = \frac{v}{n} \tag{6.10}$$

where e = void ratio
n = porosity

Darcy's law as defined by Eq. (6.6) implies that the discharge velocity v bears a linear relationship to the hydraulic gradient i and passes through the origin as shown in Figure 6.4. Hansbo (1960), however, reported the test results for four undisturbed

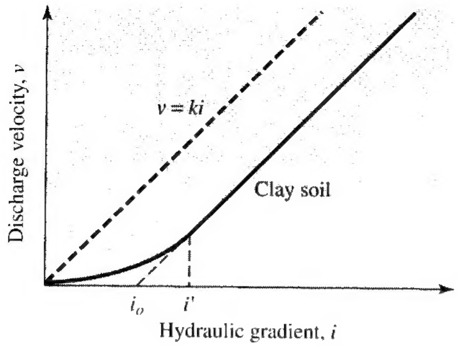

Figure 6.4
Variation of discharge velocity with hydraulic gradient in clay

natural clays. On the basis of his results, a hydraulic gradient i' (see Figure 6.4) appears to exist, at which

$$v = k(i - i_0) \qquad (\text{for } i \geq i') \tag{6.11}$$

and

$$v = ki^m \qquad (\text{for } i < i') \tag{6.12}$$

The preceding equation implies that for very low hydraulic gradients, the relationship between v and i is nonlinear. The value of m in Eq. (6.12) for four Swedish clays was about 1.5. However, several other studies refute the preceding findings. Mitchell (1976) discussed these studies in detail. Taking all points into consideration, he concluded that Darcy's law is valid.

6.3 Hydraulic Conductivity

Hydraulic conductivity is generally expressed in cm/sec or m/sec in SI units and in ft/min or ft/day in English units.

The hydraulic conductivity of soils depends on several factors: fluid viscosity, pore-size distribution, grain-size distribution, void ratio, roughness of mineral particles, and degree of soil saturation. In clayey soils, structure plays an important role in hydraulic conductivity. Other major factors that affect the permeability of clays are the ionic concentration and the thickness of layers of water held to the clay particles.

The value of hydraulic conductivity (k) varies widely for different soils. Some typical values for saturated soils are given in Table 6.1. The hydraulic conductivity of unsaturated soils is lower and increases rapidly with the degree of saturation.

Table 6.1 Typical Values of Hydraulic Conductivity of Saturated Soils

	k	
Soil type	**cm/sec**	**ft/min**
Clean gravel	100–1.0	200–2.0
Coarse sand	1.0–0.01	2.0–0.02
Fine sand	0.01–0.001	0.02–0.002
Silty clay	0.001–0.00001	0.002–0.00002
Clay	<0.000001	<0.000002

The hydraulic conductivity of a soil is also related to the properties of the fluid flowing through it by the equation

$$k = \frac{\gamma_w}{\eta}\overline{K} \tag{6.13}$$

where γ_w = unit weight of water
η = viscosity of water
$\overline{K}$ = absolute permeability

The *absolute permeability* $\overline{K}$ is expressed in units of L^2 (that is, cm^2, ft^2, and so forth).

Equation (6.13) showed that hydraulic conductivity is a function of the unit weight and the viscosity of water, which is in turn a function of the temperature at which the test is conducted. So, from Eq. (6.13),

$$\frac{k_{T_1}}{k_{T_2}} = \left(\frac{\eta_{T_2}}{\eta_{T_1}}\right)\left(\frac{\gamma_{w(T_1)}}{\gamma_{w(T_2)}}\right) \tag{6.14}$$

where k_{T_1}, k_{T_2} = hydraulic conductivity at temperatures T_1 and T_2, respectively
η_{T_1}, η_{T_2} = viscosity of water at temperatures T_1 and T_2, respectively
$\gamma_{w(T_1)}, \gamma_{w(T_2)}$ = unit weight of water at temperatures T_1 and T_2, respectively

It is conventional to express the value of k at a temperature of 20°C. Within the range of test temperatures, we can assume that $\gamma_{w(T_1)} \simeq \gamma_{w(T_2)}$. So, from Eq. (6.14)

$$k_{20°C} = \left(\frac{\eta_{T°C}}{\eta_{20°C}}\right)k_{T°C} \tag{6.15}$$

The variation of $\eta_{T°C}/\eta_{20°C}$ with the test temperature T varying from 15 to 30°C is given in Table 6.2.

Table 6.2 Variation of $\eta_{T°C}/\eta_{20°C}$

Temperature, *T* (°C)	$\eta_{T°C}/\eta_{20°C}$	**Temperature, *T* (°C)**	$\eta_{T°C}/\eta_{20°C}$
15	1.135	23	0.931
16	1.106	24	0.910
17	1.077	25	0.889
18	1.051	26	0.869
19	1.025	27	0.850
20	1.000	28	0.832
21	0.976	29	0.814
22	0.953	30	0.797

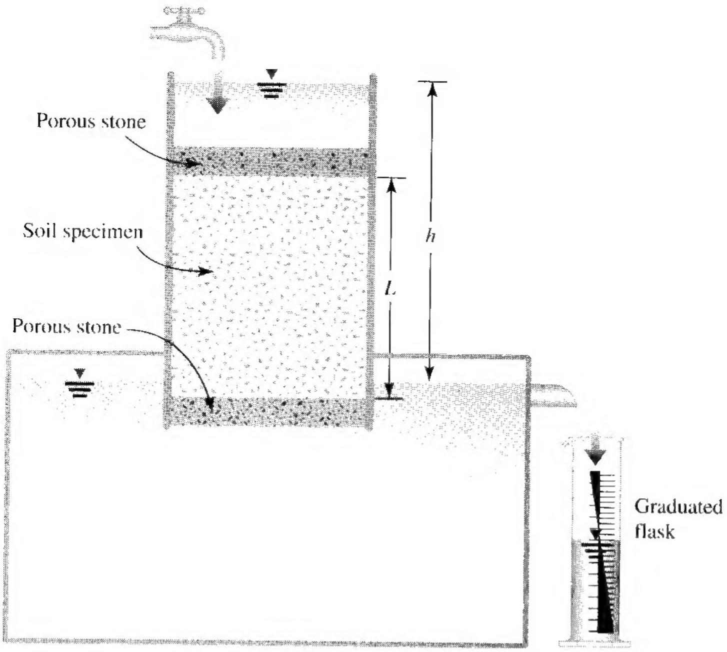

Figure 6.5 Constant-head permeability test

6.4 Laboratory Determination of Hydraulic Conductivity

Two standard laboratory tests are used to determine the hydraulic conductivity of soil — the constant-head test and the falling-head test. A brief description of each follows.

Constant-Head Test

A typical arrangement of the constant-head permeability test is shown in Figure 6.5. In this type of laboratory setup, the water supply at the inlet is adjusted in such a way that the difference of head between the inlet and the outlet remains constant during the test period. After a constant flow rate is established, water is collected in a graduated flask for a known duration.

The total volume of water collected may be expressed as

$$Q = Avt = A(ki)t \tag{6.16}$$

where Q = volume of water collected
A = area of cross section of the soil specimen
t = duration of water collection

And because

$$i = \frac{h}{L} \tag{6.17}$$

where L = length of the specimen, Eq. (6.17) can be substituted into Eq. (6.16) to yield

$$Q = A\left(k\frac{h}{L}\right)t \tag{6.18}$$

or

$$k = \frac{QL}{Aht} \tag{6.19}$$

Falling-Head Test

A typical arrangement of the falling-head permeability test is shown in Figure 6.6. Water from a standpipe flows through the soil. The initial head difference h_1 at time $t = 0$ is recorded, and water is allowed to flow through the soil specimen such that the final head difference at time $t = t_2$ is h_2.

The rate of flow of the water through the specimen at any time t can be given by

$$q = k\frac{h}{L}A = -a\frac{dh}{dt} \tag{6.20}$$

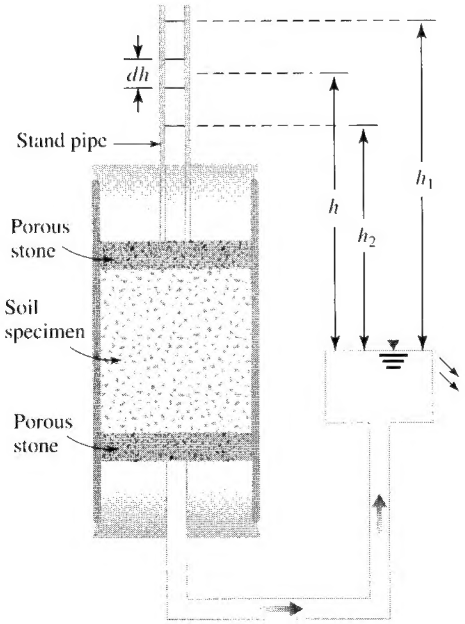

Figure 6.6 Falling-head permeability test

where q = flow rate
a = cross-sectional area of the standpipe
A = cross-sectional area of the soil specimen

Rearrangement of Eq. (6.20) gives

$$dt = \frac{aL}{Ak}\left(-\frac{dh}{h}\right) \tag{6.21}$$

Integration of the left side of Eq. (6.21) with limits of time from 0 to t and the right side with limits of head difference from h_1 to h_2 gives

$$t = \frac{aL}{Ak}\log_e \frac{h_1}{h_2}$$

or

$$k = 2.303\frac{aL}{At}\log_{10}\frac{h_1}{h_2} \tag{6.22}$$

Example 6.1

Find the flow rate in m³/sec/m length (at right angles to the cross section shown) through the permeable soil layer shown in Figure 6.7 given H = 8m, H_1 = 3m, h = 4m, L = 50m, α = 8°, and k = 0.08 cm/sec.

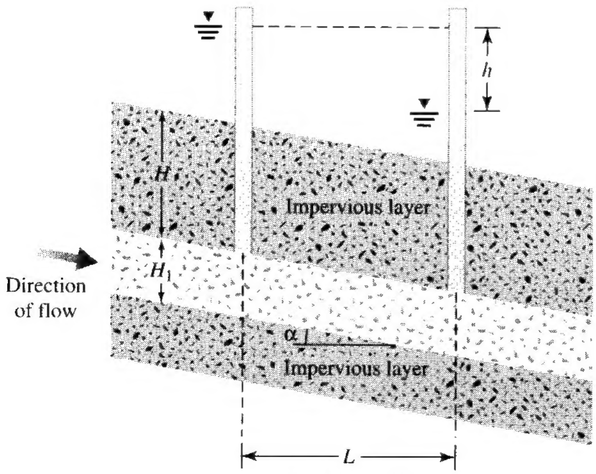

Figure 6.7 Flow-through permeable layer

Solution

$$\text{Hydraulic gradient } (i) = \frac{h}{\dfrac{L}{\cos\alpha}}$$

From Eqs. (6.17) and (6.18),

$$q = kiA = k\left(\frac{h\cos\alpha}{L}\right)(H_1\cos\alpha \times 1)$$

$$= (0.08 \times 10^{-2}\ \text{m/sec})\left(\frac{4\cos 8^\circ}{50}\right)(3\cos 8^\circ \times 1)$$

$$= 0.19 \times 10^{-3}\ \text{m}^3/\text{sec/m}$$ ■

Example 6.2

The results of a constant-head permeability test for a fine sand sample having a diameter of 150 mm and a length of 300 mm are as follows:

Constant head difference = 500 mm
Time of collection of water = 5 min
Volume of water collected = 350 cm^3
Temperature of water = 24°C

Find the hydraulic conductivity for the soil at 20°C.

Solution

For constant-head permeability test,

$$k = \frac{QL}{Aht}$$

Given that $Q = 350\ \text{cm}^3$, $L = 300$ mm, $A = (\pi/4)(150)^2 = 17671.46\ \text{mm}^2$, $h = 500$ mm, and $t = 5 \times 60 = 300$ sec, we have

change to mm^3
$\downarrow$

$$k = \frac{(350 \times 10^3) \times 300}{17671.46 \times 500 \times 300} = 3.96 \times 10^{-2}\ \text{mm/sec}$$

$$= 3.96 \times 10^{-3}\ \text{cm/sec}$$

$$k_{20} = k_{24}\frac{\eta_{24}}{\eta_{20}}$$

From Table 6.2,

$$\frac{\eta_{24}}{\eta_{20}} = 0.91$$

So, $k_{20} = (3.96 \times 10^{-3}) \times 0.91 = 3.6 \times 10^{-3}$ cm/sec. ■

Example 6.3

For a variable-head permeability test, the following are given: length of specimen = 15 in., area of specimen = 3 in.^{2}, and $k = 0.0688$ in./min. What should be the area of the standpipe for the head to drop from 25 to 12 in. in 8 min.?

Solution
From Eq. (6.22),

$$k = 2.303 \frac{aL}{At} \log_{10} \frac{h_1}{h_2}$$

$$0.0688 = 2.303 \left(\frac{a \times 15}{3 \times 8}\right) \log_{10}\left(\frac{25}{12}\right)$$

$$a = \mathbf{0.15\ in.^2}$$ ■

Example 6.4

The hydraulic conductivity of a clayey soil is 3×10^{-7} cm/sec. The viscosity of water at 25°C is 0.0911×10^{-4} g · sec/cm^2. Calculate the absolute permeability $\overline{K}$ of the soil.

Solution
From Eq. (6.13),

$$k = \frac{\gamma_w}{\eta}\overline{K} = 3 \times 10^{-7} \text{ cm/sec}$$

so

$$3 \times 10^{-7} = \left(\frac{1 \text{ g/cm}^3}{0.0911 \times 10^{-4}}\right)\overline{K}$$

$$\overline{K} = \mathbf{0.2733 \times 10^{-11}\ cm^2}$$ ■

6.5 Empirical Relations for Hydraulic Conductivity

Several empirical equations for estimating hydraulic conductivity have been proposed in the past. Some of these are briefly discussed in this section.

For fairly uniform sand (that is, sand with a small uniformity coefficient), Hazen (1930) proposed an empirical relationship for hydraulic conductivity in the form

$$k\ (\text{cm/sec}) = cD_{10}^2 \tag{6.23}$$

where c = a constant that varies from 1.0 to 1.5
D_{10} = the effective size, in mm

Equation (6.23) is based primarily on Hazen's observations of loose, clean, filter sands. A small quantity of silts and clays, when present in a sandy soil, may change the hydraulic conductivity substantially.

Casagrande proposed a simple relationship for hydraulic conductivity for fine-to-medium clean sand in the form

$$k = 1.4e^2 k_{0.85} \tag{6.24}$$

where k = hydraulic conductivity at a void ratio e
$k_{0.85}$ = the corresponding value at a void ratio of 0.85

Another form of equation that gives fairly good results in estimating the hydraulic conductivity of sandy soils is based on the Kozeny-Carman equation. The derivation of this equation is not presented here. Interested readers are referred to any advanced soil mechanics book (for example, Das, 1997). An application of the Kozeny-Carman equation yields

$$k \propto \frac{e^3}{1+e} \tag{6.25}$$

where k = hydraulic conductivity at a void ratio of e. This equation can be rewritten as

$$k = C_1 \frac{e^3}{1+e} \tag{6.26}$$

where C_1 = a constant.

Mention was made at the end of Section 6.1 that turbulent flow conditions may exist in very coarse sands and gravels, and that Darcy's law may not be valid for these materials. However, under a low hydraulic gradient, laminar flow conditions usually exist. Kenney, Lau, and Ofoegbu (1984) conducted laboratory tests on granular soils in which the particle sizes in various specimens ranged from 0.074 to 25.4 mm. The uniformity coefficients, C_u, of these specimens ranged from 1.04 to 12. All permeability tests were conducted at a relative density of 80% or more. These tests showed that for laminar flow conditions,

$$\bar{K}\ (\text{mm}^2) = (0.05 \text{ to } 1)D_5^2 \tag{6.27}$$

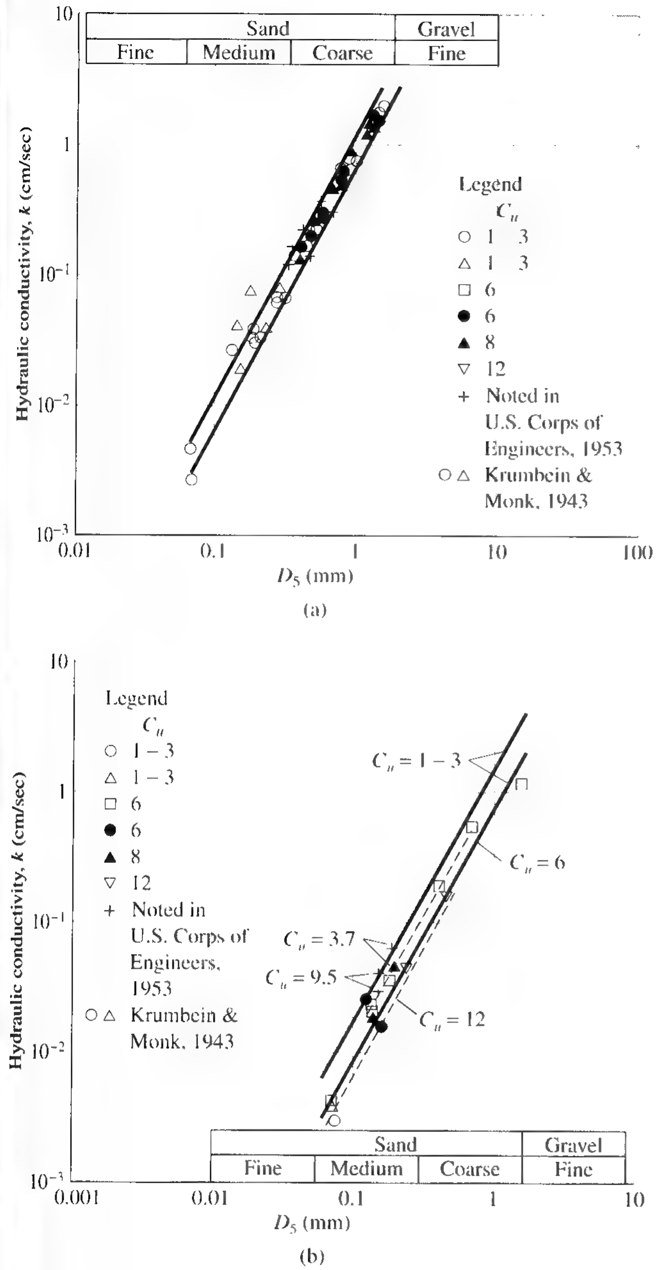

Figure 6.8
Results of permeability tests on which Eq. (6.27) is based: (a) results for $C_u = 1$–3; (b) results for $C_u > 3$ (after Kenney, Lau, and Ofoegbu, 1984)

where D_5 = diameter (mm) through which 5% of soil passes. Figures 6.8a and 6.8b show the results on which Eq. (6.27) is based.

On the basis of laboratory experiments, the U.S. Department of Navy (1971) provided an empirical correlation between k (ft/min) and D_{10}(mm) for granular soils with the uniformity coefficient varying between 2 and 12 and $D_{10}/D_5 < 1.4$. This correlation is shown in Figure 6.9.

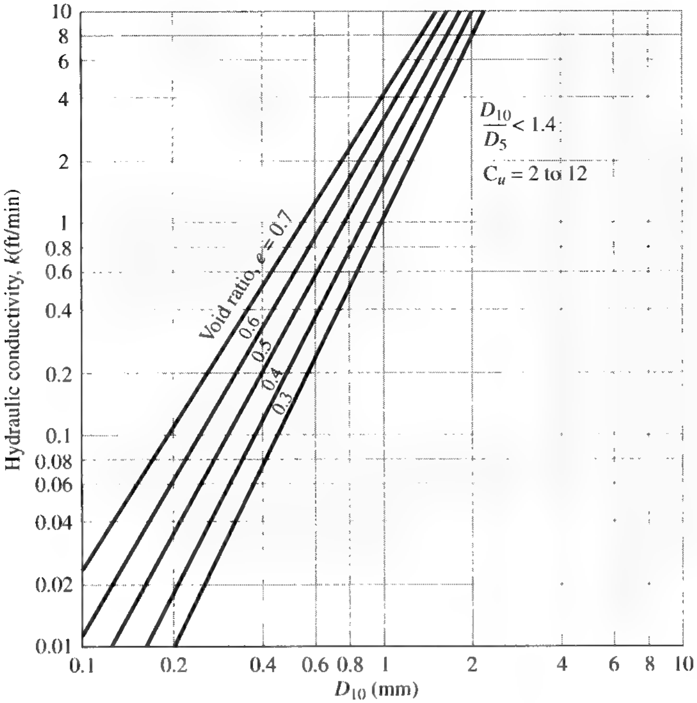

Figure 6.9 Permeability of granular soils (after U.S. Department of Navy, 1971)

According to their experimental observations, Samarasinghe, Huang, and Drnevich (1982) suggested that the hydraulic conductivity of normally consolidated clays (see Chapter 10 for definition) can be given by

$$k = C_3\left(\frac{e^n}{1+e}\right) \tag{6.28}$$

where C_3 and n are constants to be determined experimentally. This equation can be rewritten as

$$\log[k(1+e)] = \log C_3 + n \log e \tag{6.29}$$

Hence, for any given clayey soil, if the variation of k with the void ratio is known, a log-log graph can be plotted with $k(1 + e)$ against e to determine the values of C_3 and n.

Some other empirical relationships for estimating the hydraulic conductivity in sand and clayey soils are given in Table 6.3. One should keep in mind, however, that any empirical relationship of this type is for estimation only, because the magnitude of k is a highly variable parameter and depends on several factors.

Tavenas et al. (1983) also gave a correlation between the void ratio and the hydraulic conductivity of clayey soil. This correlation is shown in Figure 6.10. An important point to note, however, is that in Figure 6.10, *PI,* the plasticity index, and *CF,* the clay-size fraction in the soil, are in *fraction* (decimal) form.

Table 6.3 Empirical Relationships for Estimating Hydraulic Conductivity

Type of Soil	Source	Relationship[a]	Comments
Sand	Amer and Awad (1974)	$k = C_2 D_{10}^{2.32} C_u^{0.6} \frac{e^3}{1+e}$	
	Shahabi, Das, Tarquin (1984)	$k = 1.2 C_2^{0.735} D_{10}^{0.89} \frac{e^3}{1+e}$	Medium to fine sand
Clay	Mesri and Olson (1971)	$\log k = A' \log e + B'$	
	Taylor (1948)	$\log k = \log k_0 - \frac{e_0 - e}{C_k}$ $C_k \approx 0.5e_0$	For $e < 2.5$,

[a] D_{10} = effective size
C_u = uniformity coefficient
C_2 = a constant
k_0 = *in situ* hydraulic conductivity at void ratio e_0
k = hydraulic conductivity at void ratio e
C_k = permeability change index

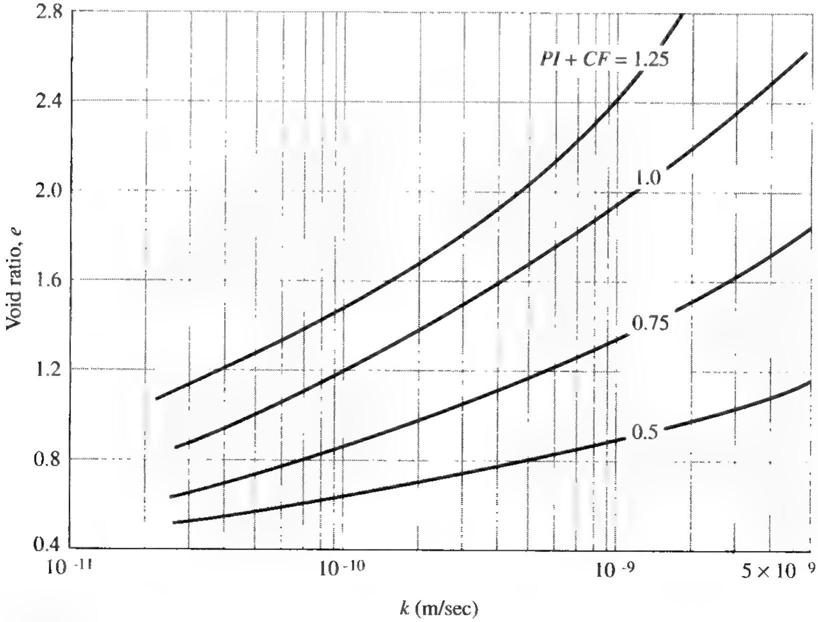

Figure 6.10 Variation of void ratio with hydraulic conductivity of clayey soils (based on Tavenas et al., 1983)

Example 6.5

The hydraulic conductivity of a sand at a void ratio of 0.8 is 0.047 cm/sec. Estimate the hydraulic conductivity of this sand at a void ratio of 0.5. Use Eq. (6.24).

Solution

From Eq. (6.24), $k = 1.4\, e^2 k_{0.85}$. Thus,

$$\frac{k_{0.8}}{k_{0.5}} = \frac{(0.8)^2}{(0.5)^2}$$

So

$$k_{0.5} = k_{0.8}\left(\frac{0.5}{0.8}\right)^2 = 0.047\left(\frac{0.5}{0.8}\right)^2$$

$$= 0.018 \text{ cm/sec}$$

■

Example 6.6

Redo Example Problem 6.5 using Eq. (6.26).

Solution

From Eq. (6.26),

$$k = C_1 \frac{e^3}{1 + e}$$

So

$$\frac{k_{0.8}}{k_{0.5}} = \frac{\left[\dfrac{0.8^3}{1 + 0.8}\right]}{\left[\dfrac{0.5^3}{1 + 0.5}\right]} = \frac{0.284}{0.083} = 3.42$$

Hence,

$$k_{0.5} = \frac{k_{0.8}}{3.42} = \frac{0.047}{3.42} \simeq 0.014 \text{ cm/sec}$$

■

Example 6.7

The void ratio and hydraulic conductivity relation for a normally consolidated clay are given below.

Void ratio	k (cm/sec)
1.2	0.6×10^{-7}
1.52	1.519×10^{-7}

Estimate the value of k for the same clay with a void ratio of 1.4.

Solution

From Eq. (6.28)

$$\frac{k_1}{k_2} = \frac{\left[\dfrac{e_1^n}{1 + e_1}\right]}{\left[\dfrac{e_2^n}{1 + e_2}\right]}$$

Substitution of $e_1 = 1.2$, $k_1 = 0.6 \times 10^{-7}$ cm/sec, $e_2 = 1.52$, $k_2 = 1.519 \times 10^{-7}$ cm/sec in the preceding equation gives

$$\frac{0.6}{1.519} = \left(\frac{1.2}{1.52}\right)^n \left(\frac{2.52}{2.2}\right)$$

or

$$n = 4.5$$

Again, from Eq. (6.28),

$$k_1 = C_3\left(\frac{e_1^n}{1 + e_1}\right)$$

$$0.6 \times 10^{-7} = C_3\left(\frac{1.2^{4.5}}{1 + 1.2}\right)$$

or

$$C_3 = 0.581 \times 10^{-7} \text{ cm/sec}$$

So

$$k = (0.581 \times 10^{-7})\left(\frac{e^{4.5}}{1 + e}\right) \text{ cm/sec}$$

Now, substituting $e = 1.4$ in the preceding equation yields

$$k = (0.581 \times 10^{-7})\left(\frac{1.4^{4.5}}{1 + 1.4}\right) = 1.1 \times 10^{-7} \text{ cm/sec}$$ ■

6.6 Directional Variation of Permeability

Most soils are not isotropic with respect to permeability. In a given soil deposit, the magnitude of k changes with respect to the direction of flow. Figure 6.11 shows a soil layer through which water flows in a direction inclined at an angle α with the vertical. Let the hydraulic conductivity in the vertical ($\alpha = 0$) and horizontal ($\alpha = 90°$) directions be k_V and k_H, respectively. The magnitudes of k_V and k_H in a given soil depend on several factors, including the method of deposition in the field. Basak (1972)

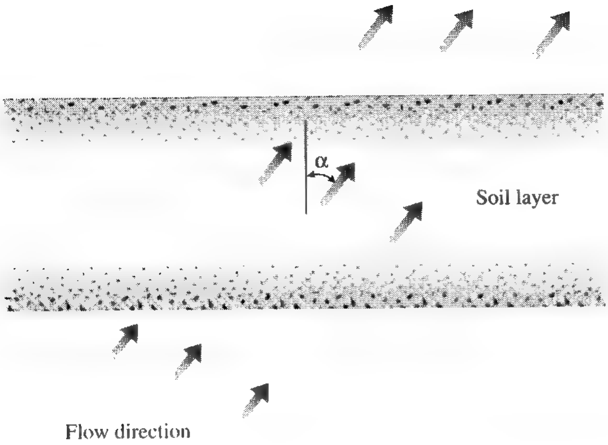

Figure 6.11 Directional variation of permeability

and Al-Tabbaa and Wood (1987) conducted laboratory tests on kaolin to determine the variation of the anisotropy ratio, $r_k = k_H/k_V$, with the void ratio. The specimens for these tests were subjected to *unidimensional* (oedometric) consolidation. The variations of r_k and void ratio (e) from the tests of Basak, and from those of Al-Tabbaa and Wood, are shown in Figure 6.12. Similar test results on a sand ($C_u = 3.5$, $e_{max} = 0.824$, and $e_{min} = 0.348$) were provided by Chapuis, Gill, and Baass (1989). The specimens for these tests were prepared by unidimensional static and dynamic compaction. The variations of r_k with e for these tests are also shown in Figure 6.12.

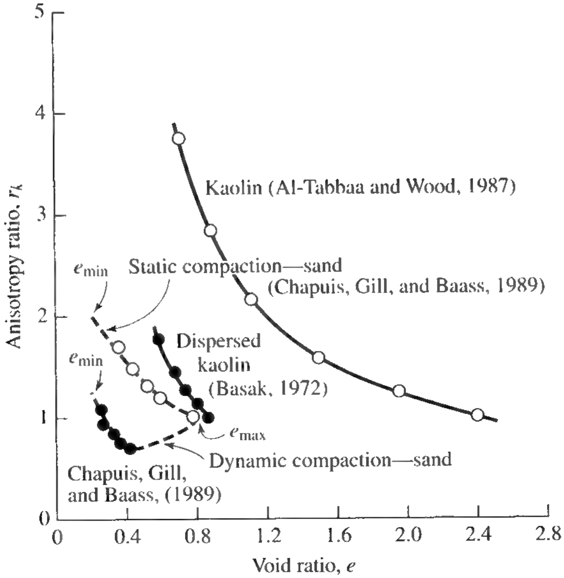

Figure 6.12 Variation of anisotropy ratio with void ratio for various soils

According to Figure 6.12, the following three general conclusions can be drawn:

1. For static compaction conditions, the magnitude of r_k decreases with the increase in void ratio.
2. For sand, the anisotropy ratio is equal to one at $e = e_{max}$.
3. For sand, with $e < e_{max}$, the magnitude of r_k is greater than one when the specimens are formed by unidimensional static compaction. However, for dynamically compacted sand, r_k may be less than one for $e < e_{max}$.

6.7 Equivalent Hydraulic Conductivity in Stratified Soil

In a stratified soil deposit where the hydraulic conductivity for flow in a given direction changes from layer to layer, an equivalent hydraulic conductivity can be computed to simplify calculations. The following derivations relate to the equivalent hydraulic conductivities for flow in vertical and horizontal directions through multilayered soils with horizontal stratification.

Figure 6.13 shows n layers of soil with flow in the *horizontal direction.* Let us consider a cross section of unit length passing through the n layer and perpendicular to the direction of flow. The total flow through the cross section in unit time can be written as

$$\begin{aligned} q &= v \cdot 1 \cdot H \\ &= v_1 \cdot 1 \cdot H_1 + v_2 \cdot 1 \cdot H_2 + v_3 \cdot 1 \cdot H_3 + \cdots + v_n \cdot 1 \cdot H_n \end{aligned} \tag{6.30}$$

where v = average discharge velocity
$v_1, v_2, v_3, \ldots, v_n$ = discharge velocities of flow in layers denoted by the subscripts

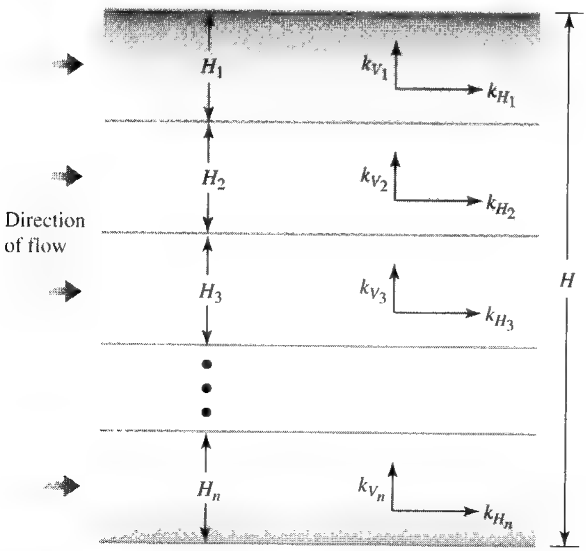

Figure 6.13 Equivalent hydraulic conductivity determination – horizontal flow in stratified soil

If $k_{H_1}, k_{H_2}, k_{H_3}, \ldots, k_{H_n}$ are the hydraulic conductivities of the individual layers in the horizontal direction and $k_{H(\mathrm{eq})}$ is the equivalent hydraulic conductivity in the horizontal direction, then, from Darcy's law,

$$v = k_{H(\mathrm{eq})} i_{\mathrm{eq}}; \quad v_1 = k_{H_1} i_1; \quad v_2 = k_{H_2} i_2; \quad v_3 = k_{H_3} i_3; \quad \ldots \quad v_n = k_{H_n} i_n$$

Substituting the preceding relations for velocities into Eq. (6.30) and noting that $i_{\mathrm{eq}} = i_1 = i_2 = i_3 = \cdots = i_n$ results in

$$k_{H(\mathrm{eq})} = \frac{1}{H}(k_{H_1} H_1 + k_{H_2} H_2 + k_{H_3} H_3 + \cdots + k_{H_n} H_n) \tag{6.31}$$

Figure 6.14 shows n layers of soil with flow in the vertical direction. In this case, the velocity of flow through all the layers is the same. However, the total head loss, h, is equal to the sum of the head losses in all layers. Thus,

$$v = v_1 = v_2 = v_3 = \cdots = v_n \tag{6.32}$$

and

$$h = h_1 + h_2 + h_3 + \cdots + h_n \tag{6.33}$$

Using Darcy's law, we can rewrite Eq. (6.32) as

$$k_{V(\mathrm{eq})}\left(\frac{h}{H}\right) = k_{V_1} i_1 = k_{V_2} i_2 = k_{V_3} i_3 = \cdots = k_{V_n} i_n \tag{6.34}$$

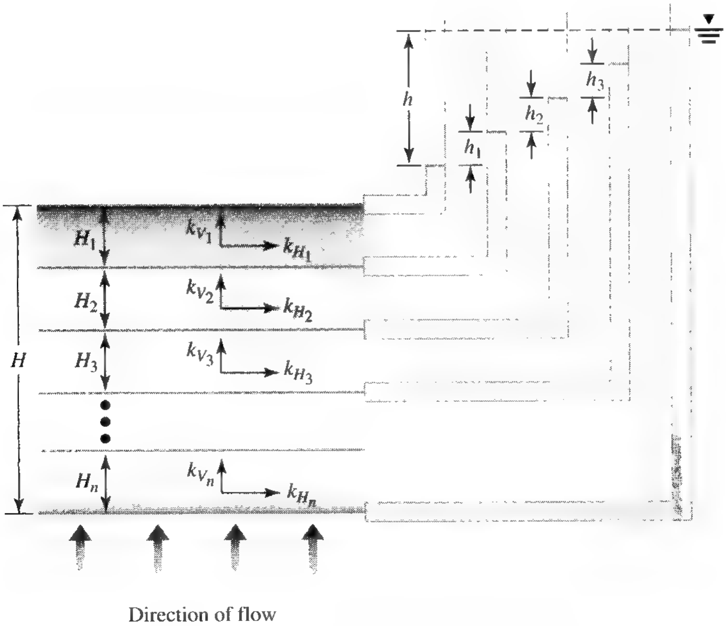

Figure 6.14 Equivalent hydraulic conductivity determination – vertical flow in stratified soil

where $k_{V_1}, k_{V_2}, k_{V_3}, \ldots, k_{V_n}$ are the hydraulic conductivities of the individual layers in the vertical direction and $k_{V(\text{eq})}$ is the equivalent hydraulic conductivity.

Again, from Eq. (6.33),

$$h = H_1 i_1 + H_2 i_2 + H_3 i_3 + \cdots + H_n i_n \tag{6.35}$$

Solving Eqs. (6.34) and (6.35) gives

$$k_{V(\text{eq})} = \frac{H}{\left(\dfrac{H_1}{k_{V_1}}\right) + \left(\dfrac{H_2}{k_{V_2}}\right) + \left(\dfrac{H_3}{k_{V_3}}\right) + \cdots + \left(\dfrac{H_n}{k_{V_n}}\right)} \tag{6.36}$$

An excellent example of naturally deposited layered soil is *varved soil,* which is a rhythmically layered sediment of coarse and fine minerals. Varved soils result from annual seasonal fluctuation of sediment conditions in glacial lakes. Figure 6.15 shows the variation of moisture content and grain-size distribution in New Liskeard, Canada, varved soil. Each varvey is about 41 to 51 mm (1.6 to 2.0 in.) thick and consists of two homogeneous layers of soil — one coarse and one fine — with a transition layer between.

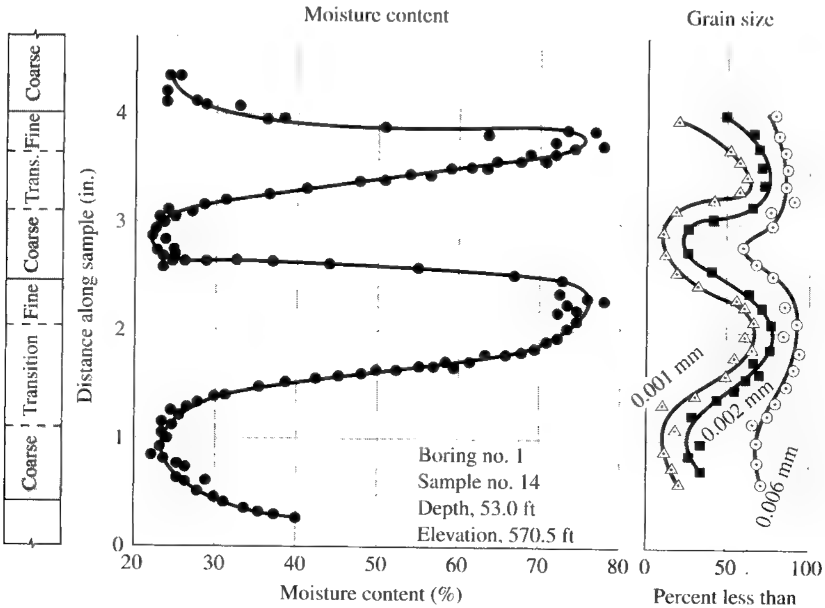

Figure 6.15 Variation of moisture content and grain-size distribution in New Liskeard varved soil. *Source:* After "Laboratory Investigation of Permeability Ratio of New Liskeard Varved Clay," by H. T. Chan and T. C. Kenney, 1973, *Canadian Geotechnical Journal, 10*(3), p. 453–472. Copyright © 1973 National Research Council of Canada. Used by permission.

Example 6.8

A layered soil is shown in Figure 6.13. Given that

- $H_1 = 1$ m $\quad k_1 = 10^{-4}$ cm/sec
- $H_2 = 1.5$ m $\quad k_2 = 3.2 \times 10^{-2}$ cm/sec
- $H_3 = 2$ m $\quad k_3 = 4.1 \times 10^{-5}$ cm/sec

estimate the ratio of equivalent permeability,

$$\frac{k_{H(\text{eq})}}{k_{V(\text{eq})}}$$

Solution

From Eq. (6.31),

$$k_{H(\text{eq})} = \frac{1}{H}(k_{H_1}H_1 + k_{H_2}H_2 + k_{H_3}H_3)$$

$$= \frac{1}{(1 + 1.5 + 2)}[(10^{-4})(1) + (3.2 \times 10^{-2})(1.5) + (4.1 \times 10^{-5})(2)]$$

$$= 107.07 \times 10^{-4} \text{ cm/sec}$$

Again, from Eq. (6.36),

$$k_{V(\text{eq})} = \frac{H}{\left(\frac{H_1}{k_{V_1}}\right) + \left(\frac{H_2}{k_{V_2}}\right) + \left(\frac{H_3}{k_{V_3}}\right)}$$

$$= \frac{1 + 1.5 + 2}{\left(\frac{1}{10^{-4}}\right) + \left(\frac{1.5}{3.2 \times 10^{-2}}\right) + \left(\frac{2}{4.1 \times 10^{-5}}\right)}$$

$$= 0.765 \times 10^{-4} \text{ cm/sec}$$

Hence,

$$\frac{k_{H(\text{eq})}}{k_{V(\text{eq})}} = \frac{107.07 \times 10^{-4}}{0.765 \times 10^{-4}} \approx \mathbf{140}$$

■

6.8 Hydraulic Conductivity of Compacted Clayey Soils

It was shown in Chapter 5 (Section 5.5) that when a clay is compacted at a lower moisture content it possesses a flocculent structure. Approximately at optimum moisture content of compaction, the clay particles have a lower degree of flocculation. A further increase in the moisture content at compaction provides a greater degree of par-

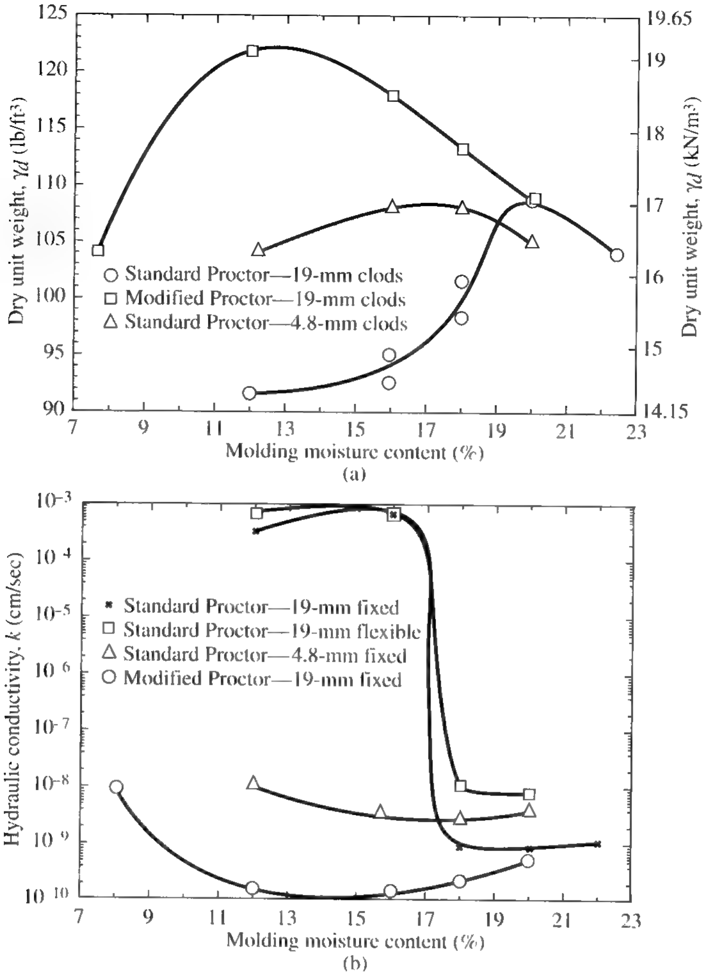

Figure 6.16 Tests on a clay soil: (a) Standard and modified Proctor compaction curves; (b) variation of k with molding moisture content. *Source:* After "Influence of Clods on Hydraulic Conductivity of Compacted Clay," by C. H. Benson and D. E. Daniel, 1990, *Journal of Geotechnical Engineering, 116*(8), p. 1231–1248. Copyright © 1990 American Society of Civil Engineers. Used by permission.

ticle orientation; however, the dry unit weight decreases because the added water dilutes the concentration of soil solids per unit volume.

Figure 6.16 shows the results of laboratory compaction tests on a clay soil as well as the variation of hydraulic conductivity on the compacted clay specimens. The compaction tests and thus the specimens for hydraulic conductivity tests were prepared from clay clods that were 19 mm and 4.8 mm. From the laboratory test results shown, the following observations can be made:

1. For similar compaction effort and molding moisture content, the magnitude of k decreases with the decrease in clod size.

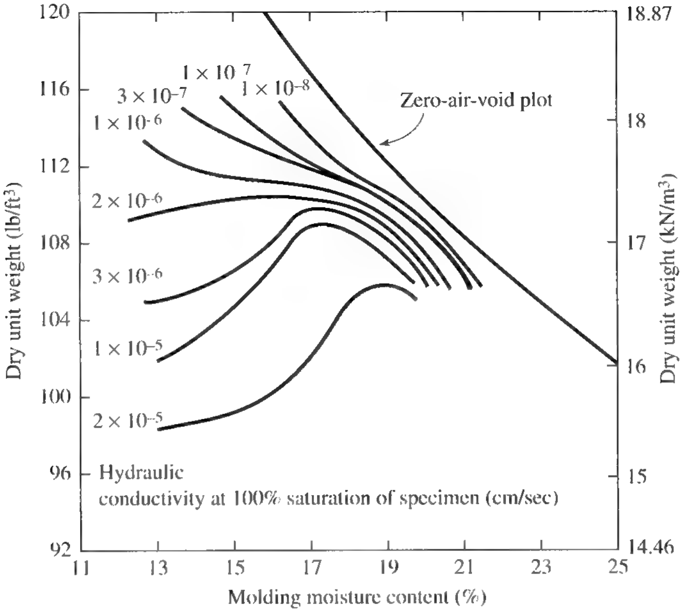

Figure 6.17 Contours of hydraulic conductivity for a silty clay. *Source:* After "Permeability of Compacted Clay," by J. K. Mitchell, D. R. Hooper, and R. B. Campenella, 1965, *Journal of the Soil Mechanics and Foundations Division, 91* (SM4), p. 41–65. Copyright © 1965 American Society of Civil Engineers. Used by permission.

2. For a given compaction effort, the hydraulic conductivity decreases with the increase in molding moisture content, reaching a minimum value at about the optimum moisture content (that is, approximately where the soil has a higher unit weight with the clay particles having a lower degree of flocculation). Beyond the optimum moisture content, the hydraulic conductivity increases slightly.
3. For similar compaction effort and dry unit weight, a soil will have a lower hydraulic conductivity when it is compacted on the wet side of the optimum moisture content. This fact is further illustrated in Figure 6.17, which shows a summary of hydraulic conductivity test results on a silty clay (Mitchell, Hooper and Campanella, 1965).

6.9 Considerations for Hydraulic Conductivity of Clayey Soils in Field Compaction

In some compaction work in clayey soils, the compaction must be done in a manner so that a certain specified upper level of hydraulic conductivity of the soil is achieved. Examples of such works are compaction of the core of an earth dam and installation of clay liners in solid-waste disposal sites.

To prevent groundwater pollution from leachates generated from solid-waste disposal sites, the U.S. Environmental Protection Agency (EPA) requires that clay

liners have a hydraulic conductivity of 10^{-7} cm/sec or less. To achieve this value, the contractor must ensure that the soil meets the following criteria (Environmental Protection Agency, 1989):

1. The soil should have at least 20% fines (fine silt and clay-size particles).
2. The plasticity index *(PI)* should be greater than 10. Soils that have a *PI* greater than about 30 are difficult to work with in the field.
3. The soil should not include more than 10% gravel-size particles.
4. The soil should not contain any particles or chunks of rock that are larger than 25 to 50 mm (1 to 2 in.).

In many instances, the soil found at the construction site may be somewhat nonplastic. Such soil may be blended with imported clay minerals like sodium bentonite to achieve the desired range of hydraulic conductivity. In addition, during field compaction, a heavy sheepsfoot roller can introduce larger shear strains during compaction that create a more dispersed structure in the soil. This type of compacted soil will have an even lower hydraulic conductivity. Small lifts should be used during compaction so that the feet of the compactor can penetrate the full depth of the lift.

The size of the clay clods has a strong influence on the hydraulic conductivity of a compacted clay. Hence, during compaction, the clods must be broken down mechanically to as small as possible. A very heavy roller used for compaction helps to break them down.

Bonding between successive lifts is also an important factor; otherwise, permeant can move through a vertical crack in the compacted clay and then travel along the interface between two lifts until it finds another crack, as is schematically shown in Figure 6.18. Bonding can substantially reduce the overall hydraulic conductivity of a compacted clay. An example of poor bonding was seen in a trial pad construction in Houston in 1986. The trial pad was 0.91 m (3 ft) thick and built in six, 15.2 mm (6 in.) lifts. The results of the hydraulic conductivity tests for the compact soil from the trial pad are given in Table 6.4. Note that although the laboratory-determined values of k for various lifts are on the order of 10^{-7} to 10^{-9} cm/sec, the actual overall value of k increased to the order of 10^{-4}. For this reason, scarification and control of

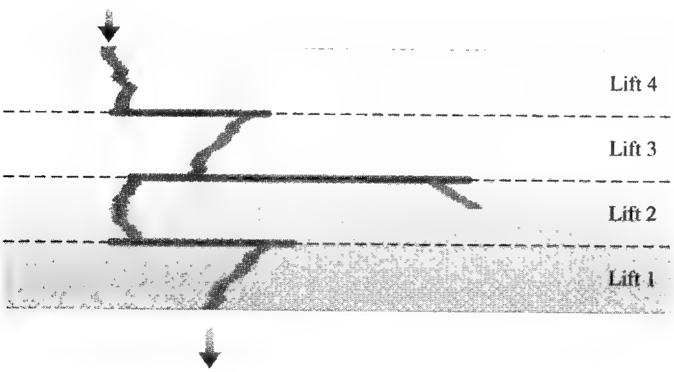

Figure 6.18 Pattern of flow through a compacted clay with improper bonding between lifts (after Environmental Protection Agency, 1989)

Table 6.4 Hydraulic Conductivity from Houston Liner Tests*

Location	Sample	Laboratory k (cm/sec)
Lower lift	76 mm (≈3 in.) tube	4×10^{-9}
Upper lift	76 mm (≈3 in.) tube	1×10^{-9}
Lift interface	76 mm (≈3 in.) tube	1×10^{-7}
Lower lift	Block	8×10^{-5}
Upper lift	Block	1×10^{-8}
Actual overall $k = 1 \times 10^{-4}$ cm/sec		

*After Environmental Protection Agency (1989)

the moisture content after compaction of each lift are extremely important in achieving the desired hydraulic conductivity.

6.10 Moisture Content—Unit Weight Criteria for Clay Liner Construction

As mentioned in Section 6.9, for construction of clay liners for solid-waste disposal sites, the compacted clay is required to have a hydraulic conductivity of 10^{-7} cm/sec or less. Daniel and Benson (1990) developed a procedure to establish the moisture content – unit weight criteria for clayey soils to meet the hydraulic conductivity requirement. Following is a step-by-step procedure to develop the criteria:

1. Conduct *modified, standard,* and *reduced* Proctor tests to establish the dry unit weight versus molding moisture content relationships (Figure 6.19a). Modified and standard Proctor tests were discussed in Chapter 5. The *reduced* Proctor test is similar to the standard Proctor test, except the hammer is dropped only 15 times per lift instead of the usual 25 times. Modified, standard, and reduced Proctor efforts represent, respectively, the upper, medium, and minimum levels of compaction energy for a typical clayey soil liner.
2. Conduct permeability tests on the compacted soil specimens (from step 1), and plot the results, as shown in Figure 6.19b. In this figure, also plot the maximum allowable value of k (that is, k_{all}).
3. Replot the dry unit weight–moisture content points (Figure 6.19c) with different symbols to represent the compacted specimens with $k > k_{all}$ and $k \leq k_{all}$.
4. Plot the acceptable zone for which k is less than or equal to k_{all} (Figure 6.19c).

6.11 Permeability Test in the Field by Pumping from Wells

In the field, the average hydraulic conductivity of a soil deposit in the direction of flow can be determined by performing pumping tests from wells. Figure 6.20a shows a case where the top permeable layer, whose hydraulic conductivity has to be determined, is unconfined and underlain by an impermeable layer. During the test, water is pumped out at a constant rate from a test well that has a perforated casing. Several observation wells at various radial distances are made around the test well.

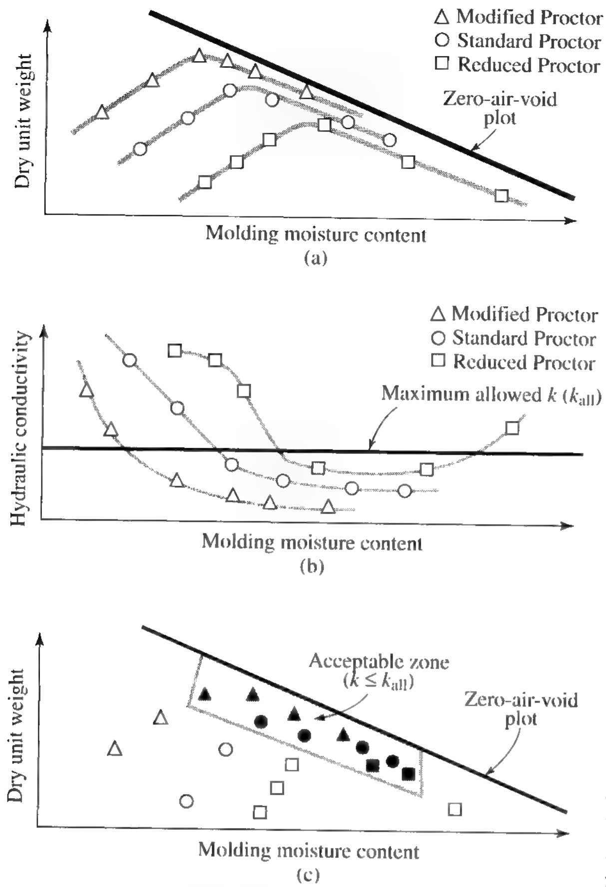

Figure 6.19
(a) Proctor curves; (b) variation of hydraulic conductivity of compacted specimens; (c) determination of acceptable zone. Source: After "Water Content-Density Criteria for Compacted Soil Liners," by D. E. Daniel and C. H. Benson, 1990, *Journal of Geotechnical Engineering, 116*(12), pp. 1811–1830. Copyright © 1990 American Society of Civil Engineers. Used by permission.

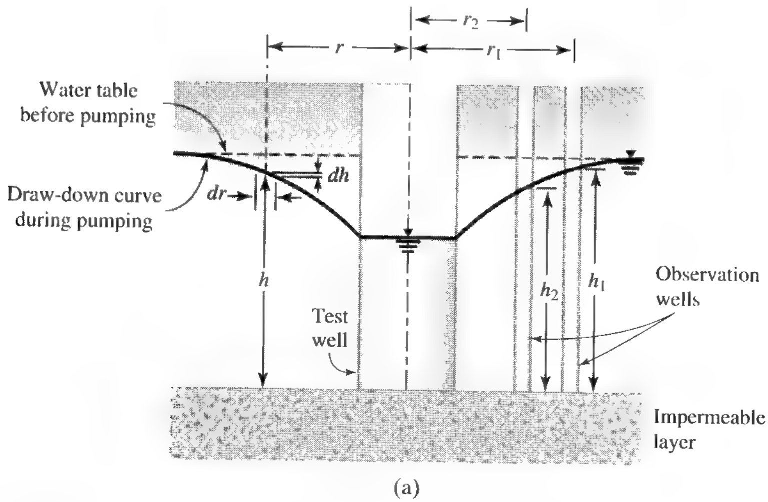

Figure 6.20 (a) Pumping test from a well in an unconfined permeable layer underlain by an impermeable stratum. Figure 6.20 continued on page 167.

Continuous observations of the water level in the test well and in the observation wells are made after the start of pumping, until a steady state is reached. The steady state is established when the water level in the test and observation wells becomes constant. The expression for the rate of flow of groundwater into the well, which is equal to the rate of discharge from pumping, can be written as

$$q = k\left(\frac{dh}{dr}\right)2\pi rh \tag{6.37}$$

or

$$\int_{r_2}^{r_1} \frac{dr}{r} = \left(\frac{2\pi k}{q}\right)\int_{h_2}^{h_1} h\,dh$$

Thus,

$$k = \frac{2.303q\log_{10}\left(\frac{r_1}{r_2}\right)}{\pi(h_1^2 - h_2^2)} \tag{6.38}$$

From field measurements, if q, r_1, r_2, h_1, and h_2 are known, the hydraulic conductivity can be calculated from the simple relationship presented in Eq. (6.38). This equation can also be written as

$$k\ (\text{cm/sec}) = \frac{2.303q\log_{10}\left(\frac{r_1}{r_2}\right)}{14.7\pi(h_1^2 - h_2^2)} \tag{6.39}$$

where q is in gpm and h_1 and h_2 are in ft.

Figure 6.20b shows the drawdown versus distance plots for a field pumping test in a deposit of coarse to fine sand, as reported by Ahmad, Lacroix, and Steinback (1975). For this test, the depth of the test well = 100 ft and q = 1515 gpm. From the plot, if we assume that steady state was reached at time t = 6064 min, we can calculate the hydraulic conductivity as follows:

r^2 (ft^2)	r (ft)	Drawdown (ft)	h (ft)
1,000	31.6	5	100 − 5 = 95
10,000	100	3.5	100 − 3.5 = 96.5

From Eq. (6.39),

$$k = \frac{(2.303)(1515)\log\left(\frac{100}{31.6}\right)}{(14.7)(\pi)(96.5^2 - 95^2)} = 0.132\ \text{cm/sec}$$

The average hydraulic conductivity for a confined aquifer can also be determined by conducting a pumping test from a well with a perforated casing that penetrates the full depth of the aquifer and by observing the piezometric level in a number of observation wells at various radial distances (Figure 6.21). Pumping is continued at a uniform rate q until a steady state is reached.

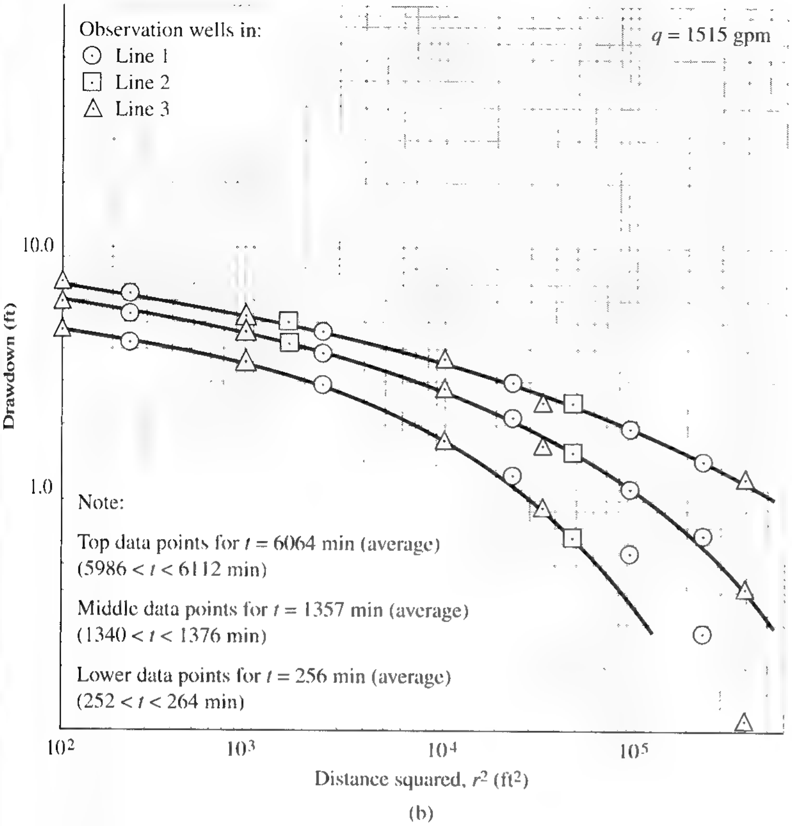

Figure 6.20 *(continued)* (b) Plot of drawdown versus r^2 in a field pumping test (based on Ahmad, Lacroix, and Steinback, 1975)

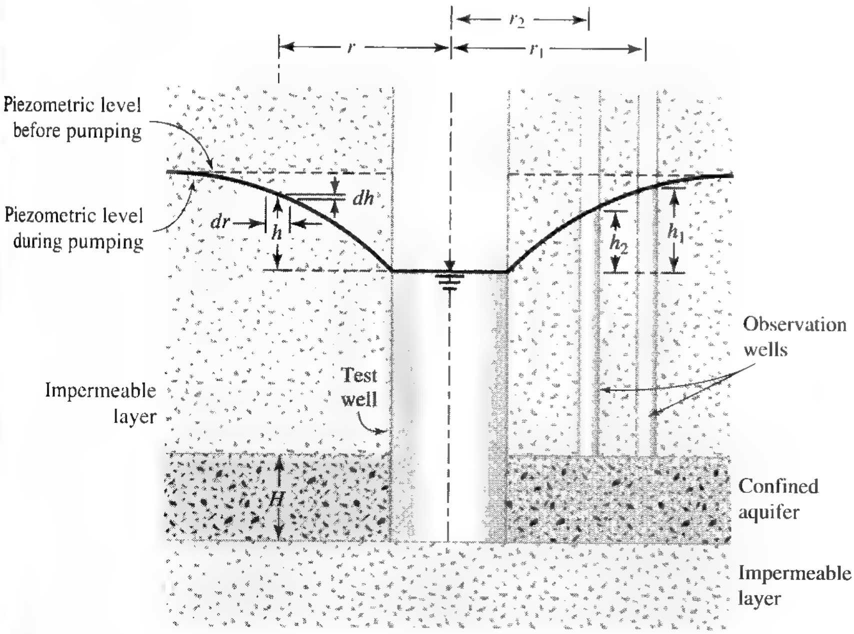

Figure 6.21
Pumping test from a well penetrating the full depth in a confined aquifer

Because water can enter the test well only from the aquifer of thickness H, the steady state of discharge is

$$q = k\left(\frac{dh}{dr}\right)2\pi rH \tag{6.40}$$

or

$$\int_{r_2}^{r_1}\frac{dr}{r} = \int_{h_2}^{h_1}\frac{2\pi kH}{q}\,dh$$

This gives the hydraulic conductivity in the direction of flow as

$$k = \frac{q\log_{10}\left(\dfrac{r_1}{r_2}\right)}{2.727H(h_1 - h_2)} \tag{6.41}$$

Example 6.9

Consider the case of pumping from a well in an unconfined permeable layer underlain by an impermeable stratum (see Figure 6.20a). Given that

- $q = 0.74\ \mathrm{m^3/min}$
- $h_1 = 6$ m at $r_1 = 60$ m
- $h_2 = 5.2$ m at $r_2 = 30$ m

calculate the hydraulic conductivity (in m/min) of the permeable layer.

Solution

From Eq. (6.38),

$$k = \frac{2.303q\log_{10}\left(\dfrac{r_1}{r_2}\right)}{\pi(h_1^2 - h_2^2)} = \frac{(2.303)(0.74)\log_{10}\left(\dfrac{60}{30}\right)}{\pi(6^2 - 5.2^2)} = \mathbf{0.018\ ft/min}$$ ■

6.12 In Situ *Hydraulic Conductivity of Compacted Clay Soils*

Daniel (1989) provided an excellent review of nine methods to estimate the *in situ* hydraulic conductivity of compacted clay layers. Three of these methods are described.

Boutwell Permeameter

A schematic diagram of the Boutwell permeameter is shown in Figure 6.22. A hole is first drilled and a casing is placed in it (Figure 6.22*a*). The casing is filled with water

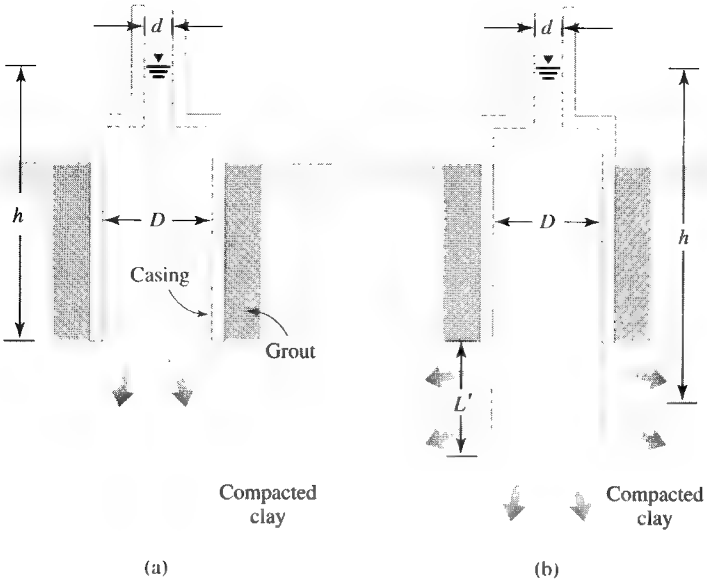

Figure 6.22 Permeability test with Boutwell permeameter

and a falling-head permeability test is conducted. Based on the test results, the hydraulic conductivity k_1 is calculated as

$$k_1 = \frac{\pi d^2}{\pi D(t_2 - t_1)} \ln\left(\frac{h_1}{h_2}\right) \tag{6.42}$$

where d = diameter of the standpipe
D = diameter of the casing
h_1 = head at time t_1
h_2 = head at time t_2

After the hydraulic conductivity is determined, the hole is deepened by augering, and the permeameter is reassembled as shown in Figure 6.22*b*. A falling-head hydraulic conductivity test is conducted again. The hydraulic conductivity is calculated as

$$k_2 = \frac{A'}{B'} \ln\left(\frac{h_1}{h_2}\right) \tag{6.43}$$

where

$$A' = d^2\left\{\ln\left[\frac{L'}{D} + \sqrt{1 + \left(\frac{L'}{D}\right)^2}\right]\right\} \tag{6.44}$$

$$B' = 8D\frac{L'}{D}(t_2 - t_1)\left\{1 - 0.562 \exp\left[-1.57\left(\frac{L'}{D}\right)\right]\right\} \tag{6.45}$$

The anisotropy with respect to permeability is determined by referring to Figure 6.23, which is a plot of k_2/k_1 versus m ($m = \sqrt{k_H/k_V}$) for various values of

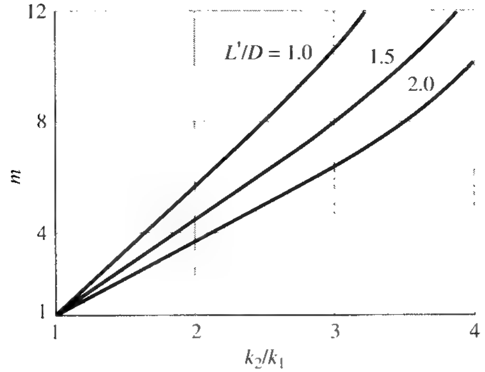

Figure 6.23 Variation of k_2/k_1 with m [Eq. (6.46)]

L'/D. Figure 6.23 can be used to determine m using the experimental values of k_2/k_1 and L'/D. The plots in this figure are determined from

$$\frac{k_2}{k_1} = \frac{\ln[(L'/D) + \sqrt{1 + (L'/D)^2}]}{\ln[(mL'/D) + \sqrt{1 + (mL'/D)^2}]} m \tag{6.46}$$

Once m is determined, we can calculate

$$k_H = mk_1 \tag{6.47}$$

and

$$k_V = \frac{k_1}{m} \tag{6.48}$$

Constant-Head Borehole Permeameter

Figure 6.24 shows a constant-head borehole permeameter. In this arrangement a constant head h is maintained by supplying water, and the rate of flow q is measured. The hydraulic conductivity can be calculated as

$$k = \frac{q}{r^2\sqrt{R^2 - 1}[F_1 + (F_2/A'')]} \tag{6.49}$$

where

$$R = \frac{h}{r} \tag{6.50}$$

$$F_1 = \frac{4.117(1 - R^2)}{\ln(R + \sqrt{R^2 - 1}) - [1 - (1/R^2)]^{0.5}} \tag{6.51}$$

$$F_2 = \frac{4.280}{\ln(R + \sqrt{R^2 - 1})} \tag{6.52}$$

$$A'' = \frac{1}{2}\alpha r \tag{6.53}$$

Typical values of α range from 0.002 to 0.01 cm^{-1} for fine-grained soil.

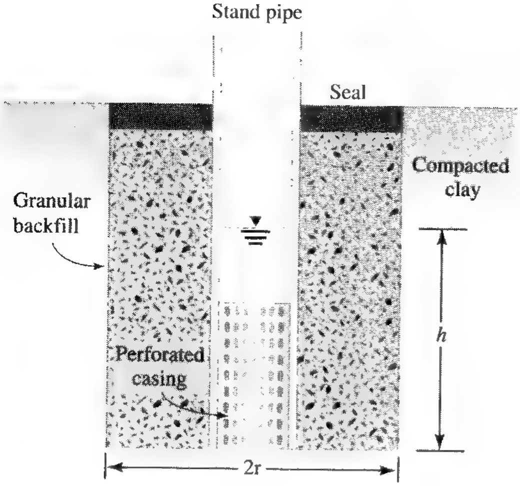

Figure 6.24
Borehole test with constant water level

Porous Probes

Porous probes (Figure 6.25) are pushed or driven into the soil. Constant- or falling-head permeability tests are performed. The hydraulic conductivity is calculated as follows:

The constant head is given by

$$k = \frac{q}{Fh} \tag{6.54}$$

The falling head is given by

$$k = \frac{\pi d^2/4}{F(t_2 - t_1)} \ln\left(\frac{h_1}{h_2}\right) \tag{6.55}$$

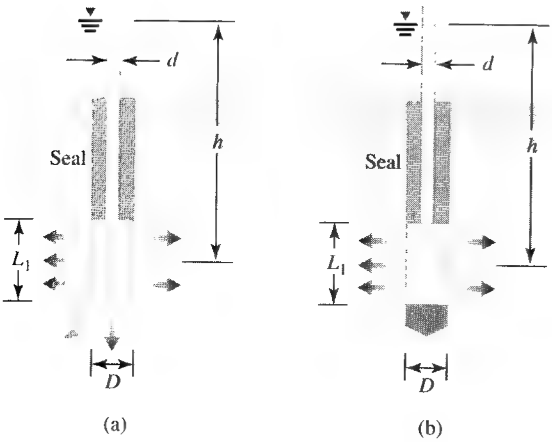

Figure 6.25 Porous probe: (a) test with permeable base; (b) test with impermeable base

For probes with permeable bases (Figure 6.25*a*),

$$F = \frac{2\pi L_1}{\ln[(L_1/D) + \sqrt{1 + (L_1/D)^2}]} \tag{6.56}$$

For probes with impermeable bases (Figure 6.25*b*),

$$F = \frac{2\pi L_1}{\ln[(L_1/D) + \sqrt{1 + (L_1/D)^2}]} - 2.8D \tag{6.57}$$

6.13 Summary and General Comments

In this chapter, we discussed Darcy's law, definition of hydraulic conductivity, laboratory determination of hydraulic conductivity and the empirical relations for it, and field determination of hydraulic conductivity of various types of soil. Hydraulic conductivity of various soil layers is highly variable. The empirical relations for hydraulic conductivity should be used as a general guide for all practical considerations. The accuracy of the values of k determined in the laboratory depends on several factors:

1. Temperature of the fluid
2. Viscosity of the fluid
3. Trapped air bubbles present in the soil specimen
4. Degree of saturation of the soil specimen
5. Migration of fines during testing
6. Duplication of field conditions in the laboratory

The hydraulic conductivity of saturated cohesive soils can also be determined by laboratory consolidation tests. (See Example 10.10.) The actual value of the hydraulic conductivity in the field may also be somewhat different than that obtained in the laboratory because of the nonhomogeneity of the soil. Hence, proper care should be taken in assessing the order of the magnitude of k for all design considerations.

Problems

6.1 A permeable soil layer is underlain by an impervious layer, as shown in Figure 6.26. With $k = 4.8 \times 10^{-3}$ cm/sec for the permeable layer, calculate the rate of seepage through it in m^3/hr/m width if $H = 3$ m and $\alpha = 5°$.

6.2 Refer to Figure 6.27. Find the flow rate in m^3/sec/m length (at right angles to the cross section shown) through the permeable soil layer. Given $H = 6$ m, $H_1 = 2.5$ m, $h = 2.8$ m, $L = 40$ m, $\alpha = 10°$, $k = 0.05$ cm/sec.

6.3 Refer to the constant-head arrangement shown in Figure 6.5. For a test, the following are given:
- $L = 24$ in.
- A = area of the specimen = 4 in.^{2}
- Constant head difference = h = 30 in.
- Water collected in 3 min = 25.1 in.^{3}

Calculate the hydraulic conductivity (in./min).

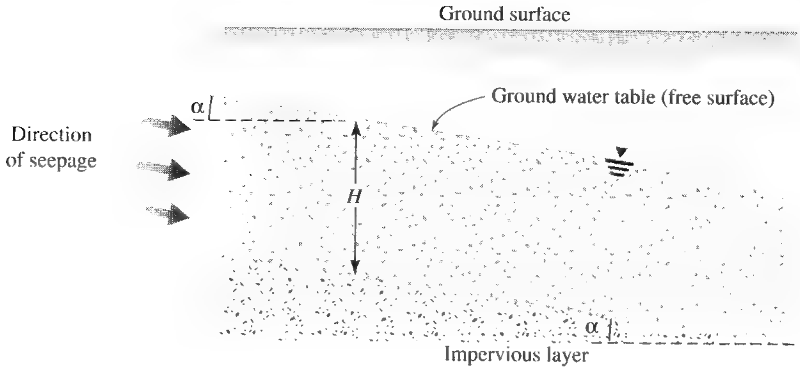

Figure 6.26

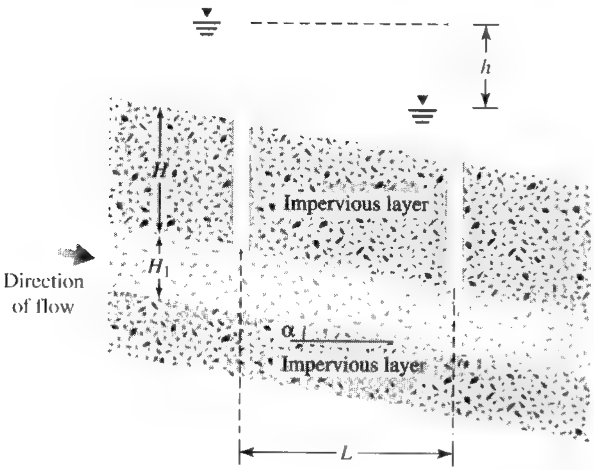

Figure 6.27

6.4 In a constant-head permeability test in the laboratory, the following are given: $L = 300$ mm and $A = 110\ \text{cm}^2$. If the value of $k = 0.02$ cm/sec and a flow rate of 140 cm^3/min must be maintained through the soil, what is the head difference, h, across the specimen? Also, determine the discharge velocity under the test conditions.

6.5 Refer to Figure 6.5. For a constant-head permeability test in a sand, the following are given:

- $L = 400$ mm
- $A = 135\ \text{cm}^2$
- $h = 450$ mm
- Water collected in 3 min $= 640\ \text{cm}^3$
- Void ratio of sand $= 0.54$

Determine the

a. Hydraulic conductivity, k (cm/sec)
b. Seepage velocity

6.6 For a variable-head permeability test, the following are given:
- Length of the soil specimen = 20 in.
- Area of the soil specimen = 2.5 in.^{2}
- Area of the standpipe = 0.15 in.^{2}
- Head difference at time $t = 0$ is 30 in.
- Head difference at time $t = 8$ min is 16 in.

a. Determine the hydraulic conductivity of the soil (in./min)
b. What is the head difference at time $t = 6$ min?

6.7 For a variable-head test, the following are given: length of specimen = 380 mm; area of specimen = 6.5 cm^2; $k = 0.175$ cm/min. What should be the area of the standpipe for the head to drop from 650 cm to 300 cm in 8 min?

6.8 The hydraulic conductivity k of a soil is 10^{-6} cm/sec at a temperature of 28° C. Determine its absolute permeability at 20° C given that, at 20° C, $\gamma_w = 9.789$ kN/m^3 and $\eta = 1.005 \times 10^{-3}$ N.s/m^2 (Newton second per meter squared).

6.9 The hydraulic conductivity of a sand at a void ratio of 0.58 is 0.04 cm/sec. Estimate its hydraulic conductivity at a void ratio of 0.45. Use Eq. (6.24).

6.10 The following are given for a sand: porosity (n) = 0.31 and k = 0.062 cm/sec. Determine k when $n = 0.4$. Use Eq. (6.25).

6.11 The maximum dry density determined in the laboratory for a quartz sand is 1650 kg/m^3. In the field, if the relative of compaction is 90%, determine the hydraulic conductivity of the sand in the field compaction condition (given that k for the sand at the maximum dry density condition is 0.04 cm/sec and $G_s = 2.68$). Use Eq. (6.25).

6.12 For a sandy soil, the following are given:
- Maximum void ratio = 0.68
- Minimum void ratio = 0.42
- Hydraulic conductivity of sand at a relative density of 70% = 0.006 cm/sec

Determine the hydraulic conductivity of the sand at a relative density of 32%. Use Eq. (6.24).

6.13 Repeat Problem 6.12 using Eq. (6.25).

6.14 For a normally consolidated clay, the following values are given:

Void ratio, *e*	*k* (cm/sec)
0.75	1.2×10^{-6}
1.2	2.8×10^{-6}

Estimate the hydraulic conductivity of the clay at a void ratio $e = 0.6$. Use Eq. (6.28).

6.15 For a normally consolidated clay, the following values are given:

Void ratio, *e*	*k* (cm/sec)
0.95	0.2×10^{-6}
1.6	0.91×10^{-6}

Determine the magnitude of k at a void ratio of 1.1. Use Eq. (6.28).

6.16 Figure 6.28 shows three layers of soil in a tube that is 100 mm × 100 mm in cross section. Water is supplied to maintain a constant head difference of

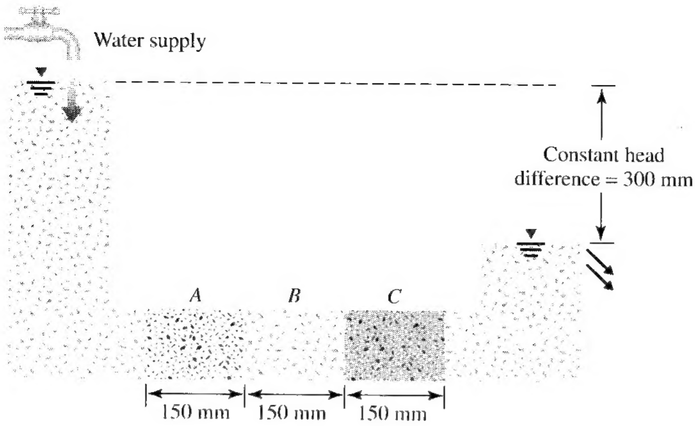

Figure 6.28

300 mm across the sample. The hydraulic conductivity of the soils in the direction of flow through them are as follows:

Soil	**k (cm/sec)**
A	10^{-2}
B	3×10^{-3}
C	4.9×10^{-4}

Find the rate of water supply in cm^3/hr.

6.17 For a clay soil, the following are given:

- Saturated unit weight = 121 lb/ft^3
- Specific gravity of soil solids (G_s) = 2.69
- Liquid limit = 46
- Plastic limit = 24
- Percent finer than 0.002 mm = 62

Estimate the hydraulic conductivity, k. Use Figure 6.10.

6.18 A layered soil is shown in Figure 6.29. Given that

- $H_1 = 1$ m $\quad k_1 = 10^{-4}$ cm/sec
- $H_2 = 1$ m $\quad k_2 = 2.8 \times 10^{-2}$ cm/sec
- $H_3 = 2$ m $\quad k_3 = 3.5 \times 10^{-5}$ cm/sec

Estimate the ratio of equivalent permeability, $k_{H(eq)}/k_{V(eq)}$.

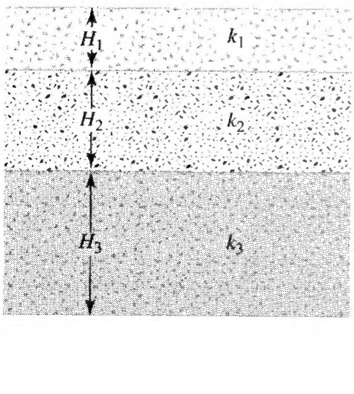

Figure 6.29

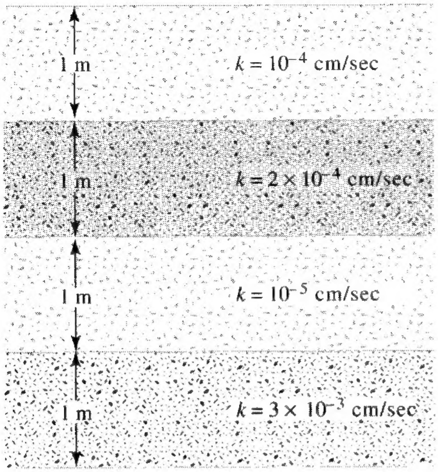

Figure 6.30

6.19 A layered soil is shown in Figure 6.30. Estimate the ratio of equivalent permeability, $k_{H(eq)}/k_{V(eq)}$.

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